Exercise 21

If $\cosh x = \frac{5}{3}$ and $x > 0$, find the values of the other hyperbolic functions at x.

Solution

Suppose that $\cosh x = \frac{5}{3}$. Then, since $\cosh^2 x - \sinh^2 x = 1$,

$$\sinh^2 x = \cosh^2 x - 1 = \left(\frac{5}{3}\right)^2 - 1 = \frac{16}{9}.$$

Take the square root of both sides.

$$\sinh x = \pm\frac{4}{3}$$

Since $x > 0$, the hyperbolic sine is positive.

$$\boxed{\sinh x = \frac{4}{3}}$$

Since $1/\sinh x = \operatorname{csch} x$,

$$\boxed{\operatorname{csch} x = \frac{3}{4}.}$$

Since $\tanh x = \sinh x / \cosh x$,

$$\boxed{\tanh x = \frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}.}$$

Since $1/\tanh x = \coth x$,

$$\boxed{\coth x = \frac{5}{4}.}$$

And since $1/\cosh x = \operatorname{sech} x$,

$$\boxed{\operatorname{sech} x = \frac{3}{5}.}$$